

QCD at Finite Density and Color Superconductivity

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1. Brief history

“Prehistoric” QCD-based calculations dealt with plasma-type phenomena like Debye screening etc [1], both for finite T and density. Early ideas about Color Superconductivity [2] were based on simple observation: unlike electrons, quarks of *different* colors are attracted to each other even by Coulomb forces. Due to Cooper instability any small attraction is enough: however the superconducting gap was only $\Delta \sim 1 \text{ MeV}$, and applicability of perturbative QCD was in doubt.

My interest was initiated by finding [3] that in the instanton liquid model even without *any* quark matter, the *ud* scalar diquarks are very deeply bound, by amount comparable to the constituent quark mass. So, phenomenological manifestations [4] of such diquarks have in fact deep dynamical roots: they follow from the same basic dynamics as the “superconductivity” of the QCD vacuum, the chiral (χ)-symmetry breaking. These spin-isospin-zero diquarks are related to pions, and should be quite robust element of nucleon (octet baryons) structure².

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²As opposed to Δ (decuplet) baryons.

Another argument for deeply bound diquarks comes from bi-color ($N_c = 2$) theory: in it the scalar diquark is degenerate with pions. By continuity from $N_c = 2$ to 3, a trace of it should exist in real QCD³.

Explicit calculations with instanton-induced forces for $N_f = 2, N_c = 3$ QCD have been made in two simultaneous⁴ papers [5,6]. Indeed, a very robust Cooper pairs and gaps $\Delta \sim 100 \text{ MeV}$ were found. From then on, the field is booming.

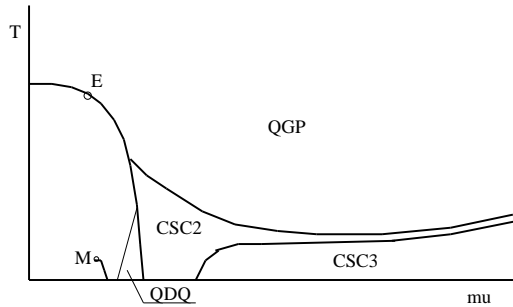
This phase (called CSC2) has the same symmetries as discussed before [2]: the χ -symmetry is restored but color group is broken by the diquark condensate, acting like Higgs VEV of the Standard Model. New variety of color superconductor, CSC3 with *Color-flavor locking* exists for 3 or more light flavors $N_f = 3$, see sect.5. At asymptotically high densities the perturbation theory must become right, see section 6. Finally we will have discussion of some outstanding issues in sect.7.

³Instanton-induced interaction strength in diquark channel is $1/(N_c - 1)$ of that for $\bar{q}\gamma_5 q$ one. It is the same at $N_c = 2$, zero for large N_c , and is exactly in between for $N_c = 3$.

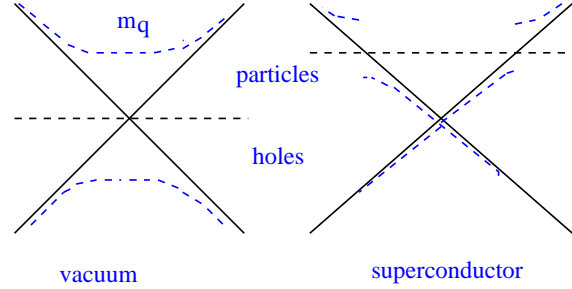
⁴Submitted to hep-ph on the same day.

2. Physics Overview

The QCD phase diagram, as we understand it today, is shown in fig.1 at the $\mu - T$ plane (baryonic chemical potential-temperature). At small T, μ there is ordinary hadronic matter with broken chiral symmetry. The point M (from “multi-fragmentation”) is the endpoint of the nuclear liquid-gas phase transition. At the (hypothetical) critical point E the first order line either continue as 2-nd order (for $m_u, m_d = 0$) or disappears (for finite masses): according to recent proposal [8] it can be found in real heavy ion collisions. QDQ (quark-diquark) phase is hypothetical [10]: I have no time to speak about it here. The main point is locations of the two superconducting phases, CSC2 and CSC3. At $T=0$ going to large μ , the χ -symmetry seem to be first recovered in CSC2, and then broken again in CSC3.



Let me then explain few major physics points. Why is there a transition from *particle-hole* to *particle-particle* pairing? The following figure (dispersion curves $\omega(k)$) for quarks in vacuum and superconductor explains it: it is better to have a gap at the surface of the Fermi sphere rather than the Dirac sea.

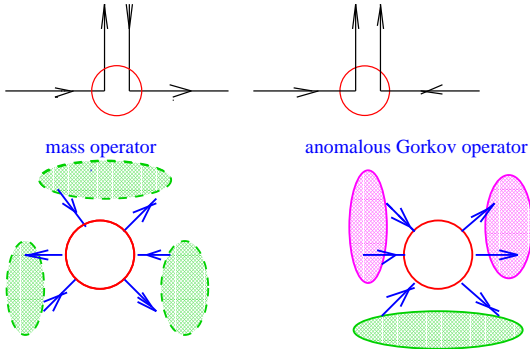


Why instantons? The reasons are: (i) They are the *strongest* non-perturbative effect known; (ii) Unlike OGE, they *do* explain quantitatively χ -symmetry breaking in vacuum; (iii) Anomaly cannot be eliminated by finite density, so tunneling leads to level crossing at the surface of the Fermi sphere as well.

Instantons create the following amusing *trialogy*: there are three attractive channels which compete: (i) the instanton-induced attraction in $\bar{q}q$ channel leading to χ -symmetry breaking. (ii) the instanton-induced attraction in qq which leads to color superconductivity. (iii) the *light-quark-induced* attraction of $\bar{I}I$, which leads to pairing of instantons into “molecules” and a Quark-Gluon Plasma (QGP) phase without *any* condensates.

How the calculations are actually done? Analytically, mostly in the mean field approximation, similar to the original BCS theory in Gorkov formulation. Total thermodynamical potential consists of “*kinetic energy*” of the quark Fermi gas, including mass operators of two types (shown in figure below). The “*potential energy*” in such approximation is the interaction Lagrangian convoluted with all possible condensates. For example, instanton-induced one with $N_f = 3$ leads to two types of diagrams shown in Fig.4,

with (a) $\langle \bar{q}q \rangle^3$ and (b) $\langle qq \rangle^2 \langle \bar{q}q \rangle$. Then one minimizes the potential over all condensates and get *gap equations*: algebra may be involved because masses/condensates are *color-flavor matrices*.



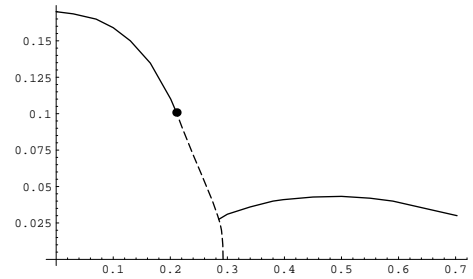
3. Two colors: a very special theory

One reason it is special is well known to lattice community: its fermionic determinant is *real* even for non-zero μ , which makes simulations possible. Early works by Karsch, Dagotto et al (of mid-80's!) made sense, but were looked at only now.

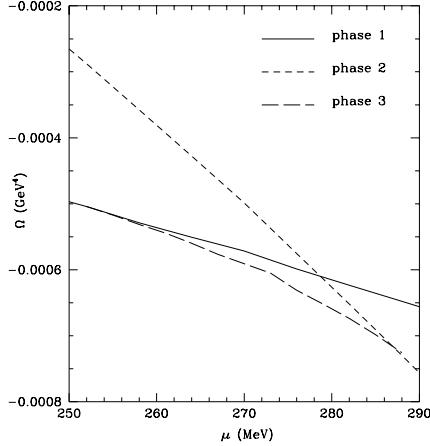
The major interest to this theory is related the so called *Pauli-Gursey symmetry*, due to which diquarks are degenerate with mesons. The χ -symmetry breaking is $SU(2N_f) \rightarrow Sp(2N_f)$, for $N_f = 2$ the coset $K = SU(4)/Sp(4) = SO(6)/SO(5) = S^5$. Those 5 massless modes are pions plus scalar diquark S and its anti-particle \bar{S} . The corresponding sigma model was worked out in [5]: for further development see [16]. As argued in [5], in this theory the critical value of transition to Color Superconductivity

is simply $\mu = m_\pi/2$, the diquark condensate is just rotated $\langle \bar{q}q \rangle$ one, and the gap is the constituent quark mass. Recent lattice works [15] and instanton liquid simulation [17] display it in great details, building confidence for other cases.

4. Two flavor QCD: the CSC2 phase

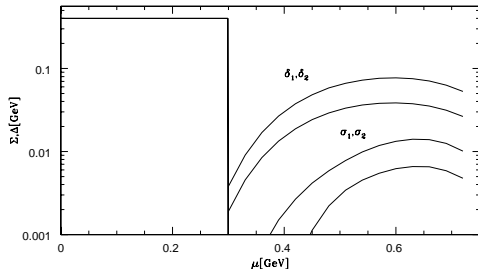


This phase diagram [11] is a rare example of calculated T - μ one: The 1st order line is dashed, and the 2nd order ones are solid lines. Most studies of this theory [5,6,12] are at $T=0$. In all these works one more possible phase (intermediate between vacuum and CSC2), *Fermi gas of constituent quarks*, with both $M, \Delta \neq 0$ - was unstable. However in last more refined calculation [10] it obtains a small window, as shown by the dashed line on the following figure. Its features are amusingly close to those of nuclear matter: but it isn't, of course: to get nucleons one should go outside the mean field. First attempted to do so in [10] was for another cluster - the $\bar{I}I$ molecules. At $T=0$ it is however only 10% correction to previous results, but is dominant as T grows.



5. $N_f = 3$ QCD: the CSC3 phase

The *color-flavor locking* [7] means that diquark condensate has the structure $\langle q_i^a C q_j^b \rangle = \bar{\Delta}_1 \delta_{ia} \delta_{bj} + \bar{\Delta}_2 \delta_{ib} \delta_{ja}$, where ij are color and ab flavor indices. It is very symmetric, reducing $SU(3)_c SU(3)_f \rightarrow SU(3)_{diagonal}$. It was verified in [7] for the OGE interaction, and for instanton-induced one in [10]: probably it is always true for that theory. Gaps δ_i and masses σ_i , following from instanton-based calculation [10], are shown as a function of μ in the following figure



Two plus strange flavor QCD ($m_s \neq 0$) was studied in several papers [10,18]. Just kinematically, us,ds Cooper pairs with zero momentum is difficult to make: for $\mu_{u,d} = \mu_s$ the momenta $p_{u,d}^F \neq p_s^F$. Instantons generate also dynamical operator $m_s(\bar{u}\bar{d})(ud)$. Resulting behavior is as shown in our first figure.

6. Asymptotically large densities

At high densities $\mu > 1\text{GeV}$ instantons are Debye-screened [9], as well as electric (Coulomb) OGE. So *magnetic* gluons overtake electric ones [13]. *Magnetically* bound Cooper pair is interesting by itself, as a rare example: one has to take care of *time delay effects* with Eliashberg eqn, etc. Angular integral leads to second log in the gap equation, leading to unusual answer: $\Delta \sim \mu \exp(-3\pi^2/\sqrt{2}g)$ which implies that the gap *grows* indefinitely with μ ⁵ and pQCD becomes finally justified. However, it is the case for huge densities, with $\mu > 10\text{GeV}$ or so.

7. Physics issues under discussion

The hadron-quark continuity. As pointed out in [19], the CSC3 phase not only has the same symmetries as hadronic matter (e.g. broken χ -symmetry), but also very similar excitations. 8 gluons become 8 *massive* vector mesons, 3×3 quarks become $8+1$ “baryons”. The 8 massless pions remain massless⁶. Furthermore,

⁵Numerical details for all densities can be found in recent work [14].

⁶Very exotic 3d objects, “super-qualitons” [20], the skyrmions made of pions are among the

photon and gluons are combined into a *massless* γ_{inside} . Can these phases be *distinquished*, and should there be *any* phase transition (in $N_f = 3$ theory)? *Is it a superconductor*, after all?

I think the answers still is “yes”. For example, if one puts a piece of CSC3 into a magnet, it may levitate: although γ_{inside} is massless, the magnet uses $\gamma_{outside}$ field and a part of it is expelled⁷.

Let me finish with few homework questions. What is the role of confinement in all these transitions? What is nuclear matter for different quark masses, anyway? Do we have other phases in between, like diquark-quark phase or (analog of) K condensation, or different crystal-like phases? Is there indeed a (remnant of) the tricritical point which we can find experimentally? And, above all, *How to do finite density calculations on the lattice?*

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excitations.

⁷The same would happen with a small piece of Weinberg/Salam vacuum, if one can make magnet with “original” (“outside”) field.